Examples of paper writing

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2021.09.29 —

Contents

Preface

These examples come from the papers (about 5 per day) arxiv pushed me every day (i subscribe the topic: Exactly Solvable and Integrable Systems) and ResearchGate recommended me. Reading these papers helped me improve my writing, although i couldn't understand most of them.

1 Introduction and Citation

- The study of discrete integrable systems has been becoming one of the most prominent branches in the theory of integrable systems in the past two decades, resulting in the establishment of many novel concepts and theories in modern mathematics, see e.g. [13]. There are several methods to constructing integrable discretisation of nonlinear differential equations. Among those, a very effective one is to construct ...

- In order to make this paper self-contained, we first briefly report the main ingredients and features of the method (see [15] and references therein). The starting point is the following theorem (see [15] for a proof), which provides the quite remarkable and well known property of the matrix solution ψ.

- This paper is concerned with

- To make this paper self-contained, in this section we briefly explain how to ... This will allow us to introduce various notational conventions. This section is closely related to [16] that we recommend for details.

- The proof here is a standard computation, see detailed examples in [16], but to make this paper self-contained, we briefly outline the argument here as well

- To help place this work into context, let us give a quick review of previous results on the ... In 1975/76 Adler [1] showed the ...

- The motivation for this work comes from Chalykh's paper [12] which generalized the KP ... A natural question to ask is

- As was remarked in [13], As in Refs. [12, 13]

- see for instance [15] and references therein.

- Quad-graph equations have attracted the interest of many researchers in the field of discrete integrable systems, see [15] for a review. This led to the development of methods for solving them, e.g. [2,13].

- have attracted a great deal of interest in ..., have attracted a lot of attention in ...

- In the recent years there is a growing interest in deriving and extending ...

- In the past two decades the research of discrete integrable systems has undergone a true development (e.g., see [1] and the references therein).

- The experimental realization of ... has opened a window towards the study of more complex ... [19-20]

- Building on our recent investigation of scalar BECs [1],

- Examples include ...
- In this paper, we develop a systematic method for computing .... While we focus on the prototypical Heisenberg XXX spin chain in the current paper, we should emphasize that our method can be generalized to any Bethe ansatz solvable spin chains.
- Let us finally mention that in the work [28], it was shown that
- Another important aspect for ... is the question of ...
- A natural question at this point is
- A particularly interesting question that arises in this context is whether
- There are not, however, a systematic procedure to construct the Lax pair or the Lax connection themselves in general. Therefore to find out if a model is integrable or not can be a big challenge.
- Many of the techniques developed to work with classical integrable models.
- Our paper is devoted to ...
- Uniqueness of domain walls with properties (a)-(c) was shown in [4] and more generally in [13].
- The following splitting lemma was pioneered in [28] and later explored in [21] in the context of vortices in $\mathbb{R}^2$.
- Among the nonlinear PDEs, of particular interest are the nonlocal integrable systems (NIS) that admit "parity-time symmetry". The activities in the field of NIS were initiated by Ablowitz and Musslimani [1], who introduced an .... Since then, the NISs have attracted much attention from both mathematics and the physical application .... Up to now, more and more nonlocal integrable equations have been established [5-7], including .... Many traditional methods, such as ..., have been used to search for exact solutions to .... Very recently, there have been some research on the nonlocal nonisospectral integrable equations. With the help of the reduction technique developed in [13], ...
- In this section, we briefly recall ... of the system (1.2). For the details one can refer to Ref. [13].
- Our motivation for the ansatz (15)-(17) is based on the following observations.
- It was found that all the elliptic solutions for the KP equation manifest themselves in terms of Riemann theta functions which are associated with algebraic curves.

- As was found/shown in [1],
- Since the middle of the 70's, it has become clear that a number of quantum anomalies have a topological nature.
- The problem of quantisation of dynamical systems has a history reaching back over a century.
- One of the key challenges that arise in ... is the ....
- The recent rapid advances in the study of integrable partial difference equations are ...
- In this review we will discuss in detail the multi-component DBSQ equations.
- We remember first some definitions and results from reference [1], where the reader can obtain more information about vector fields and differential forms over arbitrary fields.
- In the domain of mathematical modelling of physical systems the study of singularities turned out to be a most useful tool.
- Our general objective is the study of the long-time behavior of the solution $q(x,t)$.
- We would like to highlight some of the technical aspects of the manuscript.
- The main technical problem we face, mentioned in Remark 1.5, is the fact that ...
- In this paper, we introduce and solve a new exactly solvable partial integro-differential equation which is a natural spin generalization of the Benjamin-Ono (BO) equation [1,2] and which we therefore call the spin BO (sBO) equation. We present arguments that the sBO equations is not only interesting from a mathematical point of view but also for physics. We also introduce, discuss, and present results about a closely related spin generalization of the non-chiral intermediate long-wave equation recently introduced and studied by us in [3-5].
- The sBO equation is interesting for several reasons: (i) It is exactly solvable ... . (ii) ... . (iii) ... .
- Our results suggest that ..., and this opens up possibilities for several future research projects.
- Previously, we introduced advances in the research of these types of equations.
- In this paper, we are devoted to study ...

- In recent year, the development of ... has raised more interest in the discrete analysis. It is then very meaningful and also challenging to unify the discrete theory with its continuous counterpart. In our recent work [] we have successfully ....
- In order to frame our results in the context of the currently existing elliptic theory, let us review some historical milestones.

2 Definition

- The latter two matrices are assumed to have the following polynomial dependence on the complex spectral variable λ:

$$X = \Sigma + Q, Y = A + B + C, \tag{2.1}$$

where Σ is the constant, traceless, diagonal matrix, and Q is λ-independent and off-diagonal, namely $Q_{jj} = 0$, while its off-diagonal entries $Q_{jm}, j \neq m$, are generically six complex-valued functions of x and t; A is a constant matrix, while B and C can be written in terms of commutators and anti-commutators of Σ, Q and their powers.

- where E_{ij} is an $n \times n$ matrix whose (i, j) entry equals 1 while all other entries vanish.
- Here $q_1, \cdots, q_n$ are equal to zero $t = t_1$ and $t = t_2$, but otherwise arbitrary.
- for each $v \in V$, denote by $\gamma_1, \cdots, \gamma_g$ the edges incident at v in a small disk centered thereof.
- Here the notation D_{ab} stands for the product J for any two indices $a < b$.
- Denote by G_j^0 the set of matrices G_j which correspond to all variables $r_{j,i} = 1$. Then matrices G_j can be expressed in terms of G_j^0 and $r_{j,i}$ as follows:
- If any of these set is empty the corresponding product is equal to 1 by definition.
- Hamiltonian systems by definition have the form:

$$y^i = \{y^i, H\}, \tag{2.2}$$

where H is an arbitrary function, called the Hamiltonian.

- Let us denoted by Ψ the fundamental solution matrix of the equation.
- Let δ_{ij} be the Kronecker symbol and let A be a $N \times N$ matrix with elements ...

- In what follows N, Z, R, C stand, as usual, for the sets of positive integers, integers, reals and complex numbers, respectively.
- Hereafter, q and q_0 are two-component vectors, $||\cdot||$ is the standard Euclidean norm.
- Thus, hereafter, we introduce the alternative and more convenient, complex variable A defined as ...
- here and below $\bar{q}$ is the complex conjugate of q.
- Here, h refers to the height of the ...
- For compactness, we denote $f(r_{ij}) = f_{ij}$.
- the dagger denotes the matrix conjugate transpose, where the overdot indicates differentiation with respect to k,
- $A(k)$ is continuous in $0 \neq k \in \mathbb{C}^+ \cup \mathbb{R}$, is analytic in $k \in \mathbb{C}^+$, and tends to I as $k \to \infty$ from within $\mathbb{C}^+ \cup \mathbb{R}$.
- ... of the form $r^i \to f^i(r^i, \eta^i)$, where f^i is an arbitrary function of the indicated arguments.
- upper half complex plane $\mathbb{C}^+$ or the lower half complex plane $\mathbb{C}^-$.
- Let us introduce the standard partitioning $H = \begin{pmatrix} H_1 & H_2 \\ H_3 & H_4 \end{pmatrix}$ of an $n \times n$ matrix H into the $m_1 \times m_1$ matrix H_1, the $m_2 \times m_2$ matrix H_2, the $m_3 \times m_3$ matrix H_3, and the $m_4 \times m_4$ matrix H_4.
- Any $X \in \mathcal{G}$ admits the unique decomposition $X = X_1 + X_2 + X_3$ into strictly upper triangular part X_1, diagonal part X_2, and strictly lower triangular part X_3.
- where L denotes the operator which acts on a 3×3 matrix B by $LB = [L, B]$. Define $s(k)$ by ...
- In this equation, and in the rest of these lecture notes, we use the Einstein convention of summing over repeated indices.
- Let us consider a manifold M of dimension d, with local coordinates $x_i, i = 1, \cdots, d$.
- unless otherwise specified, the leading coefficients are constant.
- From now on, unless explicitly states otherwise, ...
- For notational simplicity, we write H instead of $H(L)$.
- ... the form which we call now ”the Heisenberg equations”.

- The sBO equation describes the time evolution of a square matrix-valued function $U = U(x,t)$ depending on position and time variable $x \in \mathbb{R}$ and $t \in \mathbb{R}$, respectively, and it is given by $U_t = U_x + f(U)$, where U_t is short for $\partial_t U$ etc., H is the usual Hilbert transform (which, for functions f of $x \in \mathbb{R}$ is given by ..., where $\int$ is the principal value integral), $i := \sqrt{-1}$, and $[\cdot,\cdot]$ and $\{\cdot,\cdot\}$ denote the commutator and anti-commutator of square matrices, respectively.

- Let Λ denote the lattice generated by the integrals of the basis of holomorphic differentials around this basis of cycles in C. Then the manifold $\mathbb{C}^6/\Lambda$ is the Jacobian variety of C, denoted by J. Next, for $k = 1, 2, \ldots$ define $\mathcal{A}$, the Abel map from the kth symmetric product $Sym^k(C)$ to J: ...P_1, ..., where the P_i are again points upon C. Denote the image of the kth Abel map by $W^{[k]}$, and let $a = b$. We then define ... by...

- In (3.1) ψ, S, V are $N \times N$ matrices, depending on a discrete variable $m \in \mathbb{Z}$, on a continuous variable $t \in \mathbb{R}$ and on a complex parameter λ.

3 Derivation

- By inserting in (23) the vectors (36), and by taking into account the relation $a = b$, and the matrix identity $c = d$, we end up with the expression ...

- By taking into account the expression of n_j, see (38), ...

- Because of the requirement that the basic solutions (40) be bounded functions of x, we conclude that ...

- Consequently, if A is real, large and positive then also the three wave numbers k_j, see (42), are real and large. If instead A is real and negative, and its modulus is large enough, no real wave numbers k_j exists and A does not belong to the spectrum.

- Moreover, and for the sake of simplicity, we assume that the initial values A and B are localized and bounded functions of x. With these assumptions in mind, ...

- To our purpose, and according to the formalism detailed in the previous section, it is however more convenient to choose ...

- The r.h.s of the latter defines .... Using the definition (19), the RHS of (16) becomes ...

- Given now $A \in B$ we observe from (2) that $D = C$ and $X = Y$.

- Notice that the formula (2) can alternatively be written as follows:

- Moreover a simple computation using (2.5) shows that ...

- or, after some simplification with a,
- It is then a standard computation to see that
- we obtain after a straightforward calculation that
- Plugging (2.5) in (4.4) we see that
- The previous computation shows immediately that the moment map corresponding to the group action G is given by
- We have therefore defined the monodromy map
- Therefore, denoting $\partial_t A$ by A_t we have $A = A$. Here we have used the fact that ...
- Thus (3.3) can be equivalently written as $A = A$ and further represented as
- In the course of the computation we have used that
- Recall that A, B are diagonal; we have then.
- Eliminating the variable y, we get .... Eliminating the function z from the first two equations (1.1) and (1.2) one can obtain
- The difficulty here stems from the fact that
- To find the corresponding birational change of variables, we need to keep in mind that we also have to find the relationship between the independent variables that enter geometrically as coordinates of the blowup points, so we include some details in the proof.
- Next, from the divisor matching $A = B, C = D$, and $E = F$ we get a correspondence between the coordinates of the blowup points, which in turn gives us the following relationship between
- Substituting expressions (1) for ζ then gives (2) and (3) is established directly by inverting the change of coordinates.
- Let us introduce an operator D and define ...
- Thus, to match it with $A = 4$ in (3) we should put $\lambda = 4$.
- The proof utilized the fact that
- It will be convenient to define
- On account of (2) and the locality of the subspaces (3), we see that

- Taking the derivative at $t = 0$ we obtain
- Taking the NS limit $a = \sigma, b = m, \epsilon \to 0$ we get .... We derive the $a \to 0$ limit of blowup equations from... Taking the limit as $k \to 0$ we get
- In a similar manner, we obtain ... The second parts follows in the same way. In the same way as
- It is clear that the derivative of an arbitrary function $f = f(y)$ by means of the Hamiltonian system (2) has the form
- It is clear from the above equation that ...
- Then Eq. (2) with $a = 0$, reduces to the NLS equation, namely $q_t + f(q) = 0$, to which we can further perform the rescaling $q = aq$ that leads, by dropping the tilde, to $q_t + g(q) = 0$, which is subject to the following time-independent NZBC at infinity $q = q_0$.
- it remains to express a in terms of k, which can be done as follows. Let us for brevity introduce $x = t$ ...
- Recalling now that ..., Let us recall that,
- Also, when looking at the zeros, e.g. in Fig. 3, one should keep in mind that in this system under consideration the zeros are single-valued.
- Without loss of generality, we assume that
- We limit ourselves to , We restrict our attention to , We shall restrict ourselves to , Let us restrict ourselves to , We now restrict our attention to ,
- Having at hand the exact analytical expressions for the DB solitons, we then compare them with direct ....
- At this point, it is possible to retrieve .... Recall that
- Using the identities $A = A$, we find $B = B$, whence (from where) it follows that ...
- We apply the so-called matrix triplet method to derive ...
- Putting $L = n, A = B$, where $n = m$ and B is to be determined, we compute $0 = \partial a + G + \partial f$. To cancel the ∂ terms, B should be ...
- The three-body term, written in compact form, reads
- we arrive from (1.3) at ...

- The simple structure of equation (1.1) in combination with its integrability suggests that
- The functions s, which appears in the denominators of (2.5), are ...
- To treat the second term,
- One easily checks by a direct computation that $A = 0$, in agreement with our observation that ...
- To ensure that ..., we impose the boundary condition ...
- It turns out that this is nothing but the well-known ...
- Let us note that
- The importance of such systems comes from the fact that they are completely solvable, as a consequence of Liouville's theorem.
- Due to this theorem we know that
- By using the transformation $\psi = a\phi$ and making the change of variable $x \to z := x/a$, we obtain
- In view of (take into account, considering, notice that) the decomposition $E = E_1 + E_2$, Theorem 1 establishes a rigorous connection between problems (1.1) and (1.3).
- When $y = x$, system (1.2) is nothing but the second order ...
- Here we skip the proof since it is similar to the one given in [13].
- Expanding up to second order in λ and integrating two terms by parts,
- These equations of motion can be rewritten in a more compact way, using a formal expression similar to the Hamiltonian principle of least action.
- In view of (2.1), one easily checks that (2.20) holds at $t = 0$. We will prove (2.20) by contradiction.
- The first property follows from applying Cramer's rule to (4.6) and using (4.2).
- under the assumption that ...
- Our proofs of Theorems 1.1 and 1.6 take advantage of the integrability of (1.1)-(1.2). Integrability allows one access to the ...
- We introduce the shorthand notation ... and ... to write (2.8) as ...

- We note that .... Due to this symmetry, it suffices to verify the claim for the first set of variables, i.e., it is enough to show that (2.2) follows from (2.3), (2.6), (2.8), subject to (2.4) and (2.7).

- By differentiating the first set of equations in (2.18) with respect to time, we obtain ..., where, hare and below in this section, $j = 1, \cdots, N$.

- The definition (2.16) of B_j and the relation $v = v$ imply *eq* and by using (2.17) we compute ....

- Inserting ... into ... and using ..., we find that ... is canceled by ..., and we obtain ... . To proceed, we use .. to compute ...

- For future use, it is convenient to rewrite (2.26) as ... (no need 'by') using ... and that $V(z)$ is even in the first term.

- By inserting ... and simplifying, we obtain ... using .... Since ..., we can simplify further: ...

- Thus, ... satisfies ... if and only if the following conditions are fulfilled.

- Then ... satisfies ... as long as the following conditions hold, *eq*.

- Since details of the proof are very similar to the ones of Theorem 3.1, we only explain the key differences.

- If $v = q$ and $p = u$, Eqs. (8) and (9) are reduced to, respectively, two-component cases eqs1 and eqs2.

- It remains to notice that ... to conclude the statement of the Lemma.

4 Conjunction

A word/sentence that joins words, phrases or sentences.

- Before looking at any particular reduction, let us first turn our attention to the way integrability plays an essential role in investigating the linear stability of a given solution Q.

- Before going into this investigation, we give an explicit and general construction of the matrix F by taking the advantage of the properties of ...

- Before proceeding to classify the stability ..., it is worth drawing the reader's attention to the following fact.
- Before delving into the particulars of the systems we are to study, let us defined what we mean by superintegrability. For our purposes, ...
- Before discussing ..., let us briefly describe ...
- In this section we consider reductions of the matrix Q in (10), which lead, via the compatibility condition (12), to a system of two evolution equations able to model the long wave-short wave resonant interaction. To this purpose, ..., As a consequence,
- We are now in the position to obtain the expression of the F, which corresponding to the continuous solution (23). By inserting (23) the vector (24), and by...
- Having introduced ..., we are now in the position to ...
- With the establishment of the ..., we are now in the position to ...
- In the sequel we shall use ...
- Returning now to our example, note that,
- It remains to find .... It is given in the following Lemma.
- We start by considering a homogeneous, one-dimensional ..., which is governed by ....
- Proceeding in the same way as in Section 4.2, but omitting the preliminary basis identification, we get the following result.
- The process here is analogous to the one described in Section 4.2.
- We next show
- From now on, we will suppose that
- Let us first work with / focus on
- Let us now look at the ... . After some algebra, we find that
- Let us consider now, Let us now turn our attention to, Let us turn now to, Let us now discuss,
- We now present
- The first step is to

- Having established the reduction from $N = 5$ to $N = 4$ periodic dressing chains we will taking advantage of the formalism to explore how to break the ...
- To continue, we introduce ...
- Now our goal is to establish ... To this end, we need to
- As was mentioned at the beginning of Section 1,
- To conclude this section let us note that
- In the rest of this section, we focus on
- In the remainder of this section
- We finish our discussion of ... with an explicit example of ...
- Let us start this section by introducing our notation. In what follows we deal with systems of equations
- A consequence of the above definition is the following proposition
- In this section, we aim at finding
- Since we are mostly interested in doing sth
- We begin by exploring ...
- It is also important to understand ...
- As we have seen above
- We then go on to derive ...
- Having defined the ..., we can now derive .... For this,
- Now that we have introduced the ..., we turn to the main aim of this paper, which is to explore their ...
- Continuing with the discussion, let us see how ...
- Our subsequent work is to implement ...
- With regard to the main purpose of this paper, we now briefly recall ....
- Let us next see how the energy of the perturbed solution behaves.

- To get the full power of analytic methods we have to go one step further and go to real surfaces.

  Here we go one step further and integrate over the momentum transfer squared t, ...

- Before closing this subsection,

- Now we come to state our main results.

5 Conclusion

- Although the procedure is illustrated for the systems related to P_{IV}, it is very general and can be used for other systems as well.

6 Phrase

- On the contrary

- Geometrically, ..., Topologically, ..., Algebraically, ...

- It turn out to be,

- namely, that is, that is to say, in other words, i.e., viz,

- As a matter of fact,

- make use of

- hence, therefore, thereby,

- Additionally, in addition, moreover, furthermore,

- as we shall see,

- be exclusively (the only) given as

- Analogously

- comprise, including,

- a wide variety of, plenty of, a wide/broad class of, various, a number of, a/an enormous/significant amount of, a great deal of,

- Meanwhile, in the meantime,

7 Some greatly useful website for writing

- Grammarly

  This website can help check spelling mistakes and misuse of singular and plural words in papers. You can check the entire .tex file. In addition, if you want to delete key informations in the .tex file, such as formulas, citations, references and title pages, you can use the ”Grammarly” button provided on this website (This website runs locally and will not store the tex file).

- Academic Phrasebank

  This website provides many sentence patterns for different topics.

- Linggle

  This website can provides many examples according to your search words.

- Netspeak

  This website can help complete your sentences and find similar words, which will make your sentence becomes more fluent and exact.